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Equilibrium theory under ambiguity☆

Wei He, Nicholas C. Yannelis *

Department of Economics, Henry B. Tippie College of Business, The University of Iowa, 108 John Pappajohn Business Building, Iowa City, IA 52242-1994, United States

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ABSTRACT

We extend the classical results on the Walras–core existence and equivalence to an ambiguous asymmetric information economy; that is, an economy where agents maximize Maximin Expected Utility (MEU). The interest of considering ambiguity arises from the fact that, in the presence of MEU decision making, there is no conflict between efficiency and incentive compatibility (contrary to the Bayesian decision making). Our new modeling of an ambiguous asymmetric information economy necessitates new equilibrium notions, which are always efficient and incentive compatible.

1. Introduction

Modeling the market with uncertainty is of important academic significance and realistic value in economics as most decision making is made under uncertainty. Towards this direction, the Arrow–Debreu "state contingent model" allows the state of nature of the world to be involved in the initial endowments and payoff functions, which is an enhancement of the deterministic general equilibrium model of Arrow–Debreu–McKenzie. According to Arrow–Debreu, agents make contacts ex ante (in period one) before the state of nature is realized and once the state is realized (in period two) the contract is executed and consumption takes place. The issue of incentive compatibility does not arise in this model, as all the information is symmetric. However, for the state contingent model to make sense one must assume that there is an exogenous court or government that enforces the contract ex post, otherwise agents may find it beneficial to renege. Radner (1968, 1982) extended the analysis of Arrow and Debreu by introducing asymmetric (differential) information. In particular, each agent is now characterized by his own private information, a random initial endowment, a random utility function and a prior. The private information is modeled as a partition of a finite state space and the allocation of each agent is assumed to be measurable with respect to his own private information. This means that each agent only knows the atom of his partition including the true state, but cannot distinguish those states within the same atom when making decisions. The Walrasian equilibrium notion in this model is called 'Walrasian expectations equilibrium', or WEE in short. Along this line, Yannelis (1991) proposed a core concept, which is called private core.[1]

The Walrasian expectations equilibrium and private core share some interesting properties (in fact, the Walrasian expectations equilibrium is a strict subset of the private core): without the assumption of free disposal, whenever agents are Bayesian expected utility maximizers and allocations are private information measurable, the two above notions are both Bayesian incentive compatible and private information measurable efficient (see Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994)). However, these solution concepts are only efficient in the second best sense; that is, they are only private information measurable efficient allocations and may result in a possible welfare loss (recall that from Holmstrom and Myerson (1983), we know that with the Bayesian expected utility it is not possible to have allocations which are both first best efficient and also incentive compatible). The existence of WEE in a free disposal economy can be found in Radner (1968, 1982). However, the free disposal WEE allocations may be not incentive compatible (see Glycopantis and Yannelis (2005)). Furthermore, if we require non-free disposal, then a WEE may not exist (see Einy and Shitovitz (2001)). Therefore, a natural question arises:

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* Corresponding author.
E-mail addresses: he.wei2126@gmail.com (W. He), nicholasyannelis@gmail.com (N.C. Yannelis).

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[1] For a recent treatment of general equilibrium with asymmetric information, see the books Glycopantis and Yannelis (2005) and Marakulin (2013a).

Can one find an appropriate framework in the asymmetric information economy such that the existence of equilibrium and core notions continues to hold and furthermore, these notions are both incentive compatible and first best efficient?

A crucial assumption in the frameworks of Radner (1968, 1982) and Yannelis (1991) is that agents maximize Bayesian expected utilities. Nevertheless, from Ellsberg (1961) (see also de Castro and Yannelis (2013)), there is a huge literature which criticizes the Bayesian paradigm and explores the non-expected utility theory. The maximin expected utility of Gilboa and Schmeidler (1989) is one of the successful alternatives. Indeed, recently de Castro et al. (2011, 2014) and de Castro and Yannelis (2013) applied the maximin expected utility to an asymmetric information economy with a finite number of states of nature,[2] and introduced various core and Walrasian equilibrium notions. With the maximin expected utilities, agents take into account the worst possible state that can occur and choose the best possible allocations. de Castro et al. (2011) proved that the ex ante equilibrium and core notions based on the maximin expected utility, which are called maximin expectations equilibrium (MEE) and maximin core (MC) therein, are incentive compatible in the economy without free disposal. Moreover, it is noteworthy that since the allocations are not required to be measurable with respect to agents' private information, MEE and MC allocations are also first best efficient. Therefore, the conflict between efficiency and incentive compatibility is solved in this new approach. More importantly, de Castro and Yannelis (2013) showed that the conflict of incentive compatibility and first best efficiency is inherent in the standard expected utility decision making (Bayesian) and it is resolved only when agents maximize the maximin expected utility (MEU). In particular, they proved that the MEU is a necessary condition for efficient allocations to be incentive compatible. The above work implies the fact that one has to work with MEU if the first best efficiency is desirable. As a result, a natural question arises:

Can one obtain the classical core–Walras existence and equivalence results for asymmetric information economies where agents are ambiguous (*i.e.*, MEU maximizers) and also the state space is not necessarily finite?

An affirmative answer to this question is of great importance because not only this way one develops a new equilibrium theory where there is no conflict between efficiency and incentive compatibility, but also such positive results could become the main tool for applications in other fields of economics.

The first aim of this paper is to prove the existence of the maximin expectations equilibrium and maximin core in a non-free disposal economy with countably many states of nature.[3] Since there is a countable number of states in the economy, the allocations are infinite dimensional. An advantage of the ambiguous economy modeling is that it allows us to view an asymmetric information economy as a deterministic economy with infinite dimensional commodity spaces. Thus, we can directly apply known results in the literature to obtain the existence of maximin expectations equilibrium.[4] As a corollary, we obtain that the consistency between incentive compatibility and efficiency also holds with a countable number of states.

The second aim of the current paper is to prove a core equivalence theorem for an economy with asymmetric information where agents are ambiguous (*i.e.*, maximize MEU). In a finite agent framework and complete information, Debreu and Scarf (1963) considered a sequence of replicated economy and showed that the set of non-blocked allocations in every replicated economy converges to the set of Walrasian equilibria. In Section 4, we follow the Debreu–Scarf approach and establish a similar equivalence result for an equal treatment economy with asymmetric information, a countable number of states and MEU preferences. In an atomless economy with complete information, Schmeidler (1972), Grodal (1972) and Vind (1972) improved the core–Walras equivalence theorem of Aumann (1964), by showing that if an allocation is not in the core, then it can be blocked by a non-negligible coalition with any given measure less than 1. Hervés-Beloso et al. (2005a,b) first extended this result to an asymmetric information economy with the equal treatment property and with an infinite dimensional commodity space by appealing to the finite dimensional Lyapunov's theorem. Bhowmik and Cao (2012, 2013a) obtained further extensions based on an infinite dimensional version of Lyapunov's theorem. All the above results rely on the Bayesian expected utility formulation and therefore the conflict of efficiency and incentive compatibility still holds despite the non atomic measure space of agents.[5] Our Theorem 6 is an extension of Vind's theorem to the asymmetric information economy with the equal treatment property and a countable number of states of nature, where agents behave as maximin expected utility maximizers. Thus, our new core equivalence theorem for the MEU framework, resolves the inconsistency of efficiency and incentive compatibility.

Finally, we provide two characterizations for maximin expectations equilibrium. In the complete information economy with finite agents, Aubin (1979) introduced a new approach that at a first glance seems to be different from the Debreu–Scarf; however one can show that they are essentially equivalent. Aubin considered a veto mechanism in the economy when a coalition is formed; in particular, agents are allowed to participate with any proportion of their endowments. The core notions defined by the veto mechanism, is called Aubin core and it coincides with the Walrasian equilibrium allocations. The approach of Aubin has been extended to an asymmetric information economy to characterize the Walrasian expectations equilibrium (see for example Graziano and Meo (2005), Hervés-Beloso et al. (2005b) and Bhowmik and Cao (2013a)). Another approach to characterize the Walrasian expectations equilibrium is due to Hervés-Beloso et al. (2005a,b). They showed that the Walrasian expectations equilibrium allocation cannot be privately blocked by the grand coalition in any economy with the initial endowment redistributed along the direction of the allocation itself. This approach has been extended to a pure exchange economy with an atomless measure space of agents and finitely many commodities, and an asymmetric information economy with an infinite dimensional commodity space (e.g., see Hervés-Beloso and Moreno-García (2008), Bhowmik and Cao (2013a,b)). Our Theorems 2 and 3 extend these two characterizations to the asymmetric information economy with ambiguous agents and with countably many states of nature.

The paper is organized as follows. Section 2 states the model of ambiguous asymmetric information economies with a countable number of states and discusses main assumptions.

[2] MEU is first applied to a general equilibrium model of an asymmetric information economy by Correia-da-Silva and Hervés-Beloso (2009). They proved the existence of the ex ante Walrasian equilibrium in an asymmetric information economy with maximin preferences and a finite state space. However, their setup is different from ours and they do not consider the issue of incentive compatibility; see also Correia-da-Silva and Hervés-Beloso (2012, 2014).

[3] For a general equilibrium model with countably many states, see, for example, Hervés-Beloso et al. (2009).

[4] On the contrary, one cannot readily convert an asymmetric information economy with Bayesian expected utility maximizers to an economy with infinite dimensional commodity spaces due to the restriction of the private information measurability requirement. For some papers with infinite dimensional commodity spaces, see, for example, Bewley (1972) and Podczeck and Yannelis (2008).

[5] As the work of Sun and Yannelis (2008) indicates, even with an atomless measure space of agents we cannot guarantee that WEE allocations are incentive compatible.

Section 3 introduces the maximin expectations equilibrium and maximin core and proves their existence, and contains two different characterizations of maximin expectations equilibrium by using the maximin blocking power of the grand coalition. Section 4 extends the maximin expectations equilibrium and maximin core to an economy with a continuum of agents, and interprets the asymmetric information economy with finite agents as a continuum economy with finite types. In addition, two core–Walras equivalence theorems and an extension of Vind's result are given for an asymmetric information economy with a countable number of states. Section 5 shows that maximin efficient allocations are incentive compatible in economies with finite agents and atomless economies with the equal treatment property. Section 6 collects some concluding remarks and open questions. The Appendix contains all the main proofs.

2. Ambiguous asymmetric information economy

We define an exchange economy with uncertainty and asymmetric information. The **uncertainty** is represented by a measurable space $(\Omega, \mathcal{F})$, where $\Omega = \{\omega_n\}_{n\in\mathbb{N}}$ is a countable set and $\mathcal{F}$ is the power set of Ω. Let $\mathbb{R}^l_+$ be the commodity space, and $I = \{1, 2, \ldots, s\}$ the set of agents.

For each $i \in I$, $\mathcal{F}_i$ is the σ-algebra on Ω generated by the partition Π_i of agent i, which represents the private information.[6] Let $\Pi_i(\omega)$ be the element in the partition Π_i which contains ω. Therefore, if any state $\omega \in \Omega$ is realized, then agent i can only observe the event $\Pi_i(\omega)$. The **prior** π_i of agent i is defined on $\mathcal{F}_i$ such that $\sum_{E\in\Pi_i} \pi_i(E) = 1$ and $\pi_i(E) > 0$ for every $E \in \Pi_i$. Notice that π_i is incomplete; that is, the probability of each element in the information partition Π_i is well defined, but not the probability of the event $\{\omega\}$ for every $\omega \in \Omega$. Let $u_i(\omega, \cdot): \mathbb{R}^l_+ \to \mathbb{R}_+$ be the positive **ex post utility function** of agent i at state ω from the consumption space to the positive real line, and $e_i : \Omega \to \mathbb{R}^l_+$ be i's **random initial endowment**.

Let $\mathcal{E}$ be an **ambiguous asymmetric information economy**, where

$$\mathcal{E} = \{(\Omega, \mathcal{F}); (\mathcal{F}_i, u_i, e_i, \pi_i) : i \in I = \{1, \ldots, s\}\}.$$

A **price vector** p is a nonzero function from Ω to $\mathbb{R}^l$.[7] We assume that $\triangle$ denotes the set of all price vectors, where

$$\triangle = \left\{ p \in (\mathbb{R}^l)^\infty : \left| \sum_{\omega\in\Omega} \sum_{j=1}^{l} p(\omega, j) \right| = 1 \right\},$$

and $p(\omega, j)$ is the price of the commodity j at the state ω.

There are three stages in this economy: at the ex ante stage ($t = 0$), the information partition and the economy structure are common knowledge; at the interim stage ($t = 1$), each individual i learns his private information $\Pi_i(\omega)$ which includes the true state ω, and makes his consumption plan; at the ex post stage ($t = 2$), agent i receives the endowment and consumes according to his plan.[8]

An **allocation** is a mapping x from $I \times \Omega$ to $\mathbb{R}^l_+$. For each $i \in I$, let

$$L_i = \{x_i : x_i(\omega) \in \mathbb{R}^l_+ \text{ and uniformly bounded for all } \omega \in \Omega\}$$

be the **set of all random allocations** of agent i.[9] If $x_i \in L_i$ and $p \in \triangle$, we denote $\sum_\omega \in \Omega p(\omega) \cdot x_i(\omega)$ as $p \cdot x_i$.

Suppose that x is an allocation. Then $x_i(\omega)$ is a vector in $\mathbb{R}^l_+$ for each $i \in I$, which represents the allocation at the state ω. In addition, $x_i(\omega, j)$ denotes the allocation of commodity j at the state ω. An allocation x is said to be **feasible** if $\sum_{i\in I} x_i = \sum_{i\in I} e_i$. That is, for each $\omega \in \Omega$,

$$\sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega).$$

The feasibility here indicates that the economy has no free disposal.

Assumption (E). 1. For each $i \in I$, e_i is $\mathcal{F}_i$-measurable.[10]
2. There exists some $\beta > 0$ such that for any $\omega \in \Omega$ and $1 \le j \le l$, $e_i(\omega, j) \ge \beta$.
3. There exists some $\gamma > 0$ such that for any $\omega \in \Omega$ and $1 \le j \le l$, $\sum_{i\in I} e_i(\omega, j) \le \gamma$.

Assumption (E) is about the endowment. Condition (1) says that each agent's endowment should be measurable with respect to his private information, otherwise the agent may disclose the true state from his endowment. Condition (2) implies that for every agent i, e_i is an interior point of $(\mathbb{R}^l_+)^\infty$ under the sup-norm topology. Condition (3) implies that $e_i \in L_i$; that is, the resource of the economy is limited no matter what the state is. This condition will be automatically satisfied if there are only finitely many states.[11]

Assumption (U). 1. For each $\omega \in \Omega$ and $i \in I$, $u_i(\omega, \cdot)$ is continuous, strictly increasing and concave.
2. For each $i \in I$ and $x \in \mathbb{R}^l_+$, $u_i(\cdot, x)$ is $\mathcal{F}_i$-measurable.[12]
3. For any $a \in \mathbb{R}^l_+$ and $K_0 > 0$ such that $a(j) \le K_0$ for $1 \le j \le l$, there exists some $K > 0$ such that $0 \le u_i(\omega, a) \le K$ for any $i \in I$ and $\omega \in \Omega$. Let $u_i(\omega, 0) = 0$ for all $i \in I$ and $\omega \in \Omega$.

Assumption (U) is about the utility. Conditions (1) and (2) are standard in the literature. Condition (3) basically says that agents' utility cannot be arbitrarily large with limited goods. This condition can be removed if Ω is finite: for each $i \in I$ and $\omega \in \Omega$, $u_i(\omega, a)$ is continuous at a, if a is bounded, then $u_i(\omega, \cdot)$ is bounded; since there are only finitely many states, $u_i(\omega, \cdot)$ is uniformly bounded among all ω. Moreover, the condition $u_i(\omega, 0) = 0$ means that agents have no payoff if they have no consumption.

For every agent i, his private prior may be incomplete and the allocation in L_i is not required to be $\mathcal{F}_i$-measurable. Thus, agents cannot evaluate the allocation based on the Bayesian expected utility. In the current paper, we will consider the maximin preference axiomatized by Gilboa and Schmeidler (1989).[13]

Let $\mathcal{M}_i$ be the set of all probability measures on $\mathcal{F}$ which agree with π_i on $\mathcal{F}_i$. That is,

$$\mathcal{M}_i = \{\mu : \mathcal{F} \to [0, 1] : \mu(E) = \pi_i(E),\ \forall E \in \mathcal{F}_i\}.$$

Let P_i be a nonempty and convex subset of $\mathcal{M}_i$, which is the set of priors of agent i.

We assume that agent i is ambiguous on the set P_i and will take the worst possible scenario when evaluating his payoff. In

[6] For more discussions on information partitions and σ-algebras, see, for example, Hervés-Beloso and Monteiro (2013).

[7] The vector p is said to be nonzero if p is not a constant function of value 0, but it is possible that $p(\omega) = 0$ for some ω.

[8] We consider a pure exchange economy and have no production in our model as for example in Marakulin (2013b). But the production sector can be included in the analysis and the results should still hold. For simplicity of the exposition, we have not included production.

[9] That is, $L_i = l^\infty_+$ for each $i \in I$.

[10] Clearly, if e_i is independent of ω, then it is $\mathcal{F}_i$-measurable.

[11] Since the initial endowment is bounded, the value $p \cdot e_i$ of the initial endowment e_i is finite for any agent i and price p.

[12] If u_i is state independent, then it is automatically $\mathcal{F}_i$-measurable.

[13] We can adopt the more general variational preferences axiomatized by Maccheroni et al. (2006), and all the results in Sections 3 and 4 will still go through.

particular, for any two allocations $x_i, y_i \in L_i$, agent i prefers the allocation x_i to the allocation y_i if

$$\inf_{\mu\in P_i}\sum_{\omega\in\Omega} u_i(\omega, x_i(\omega))\mu(\omega) \geq \inf_{\mu\in P_i}\sum_{\omega\in\Omega} u_i(\omega, y_i(\omega))\mu(\omega).$$

For any allocation $\{x_i\}_{i\in I}$, the **maximin ex ante utility** of agent i is:

$$V_i(x_i) = \inf_{\mu\in P_i}\sum_{\omega\in\Omega} u_i(\omega, x_i(\omega))\mu(\omega).$$

The **maximin interim utility** of agent i with allocation x_i at the state ω is

$$v_i(\omega, x_i) = \frac{1}{\pi_i(\Pi_i(\omega))}\inf_{\mu\in P_i}\sum_{\omega_1\in\Pi_i(\omega)} u_i(\omega_1, x_i(\omega_1))\mu(\omega_1).$$

We will slightly abuse the notations by writing $v_i(\omega, x_i) = v_i(E, x_i)$ for $\omega \in E \in \mathcal{F}_i$.

Remark 1. If P_i is a singleton set for each agent i, then the maximin expected utility above reduces to the standard Bayesian expected utility. If $P_i = \mathcal{M}_i$, the set of all probability measures on $\mathcal{F}$ which agree with π_i on $\mathcal{F}_i$, then it is the maximin expected utility considered in de Castro and Yannelis (2013). In the latter case, de Castro and Yannelis (2013) showed that for any two allocations $x_i, y_i \in L_i$, agent i prefers the allocation x_i to the allocation y_i if:

$$\sum_{E_i\in\Pi_i}[\inf_{\omega\in E_i} u_i(\omega, x_i(\omega))]\pi_i(E_i) \geq \sum_{E_i\in\Pi_i}[\inf_{\omega\in E_i} u_i(\omega, y_i(\omega))]\pi_i(E_i).^{14} \quad (1)$$

Remark 2. It should be noted that the asymmetric information in a Bayesian model comes from the private information measurability of allocations. For example, if allocations are not required to be private information measurable, then the framework of Radner (1968) reduces to the standard Arrow–Debreu state-contingent model. In other words, the private information measurability of allocations captures the information asymmetry in a Bayesian model. Furthermore, despite the fact that the Walrasian expectations equilibrium is incentive compatible (see Koutsougeras and Yannelis (1993)), it may be only second best efficient due to the private information measurability requirement of the allocations, which is pointed out in the current paper (see Example 2 below) as well as de Castro and Yannelis (2013).

In an ambiguity model, the information asymmetry is captured by the maximin expected utility itself. In particular, priors are defined on the information partition of each agent (while they are defined on the whole state space Ω in a Bayesian model). Thus, it is natural to relax the restriction of private information measurability of allocations in an ambiguity model. In addition, we show that the maximin expectations equilibrium is both first best efficient and incentive compatible.

The proposition below indicates that the maximin ex ante utility function satisfies several desirable properties.

Proposition 1. *If Assumption (U) holds, then V_i is increasing and concave, continuous in the sup-norm topology, and lower semicontinuous in the weak* topology.*

Proof. See Appendix. □

[14] First, we use 'inf' in these two inequalities instead of 'min' used in de Castro and Yannelis (2013), since there are infinite states here. The existence of infimum is guaranteed since the ex post utility function is nonnegative. Thus the ex ante utility V_i is well defined. Second, although (de Castro and Yannelis, 2013) only argued that these two inequalities are equivalent when there are finitely many states, this observation is still true in our context.

3. Maximin expectations equilibrium and maximin core

3.1. Existence of MEE and MC

In this section, we define the notions of maximin core (MC) and maximin expectations equilibrium (MEE).

Given a price vector p, the budget set of agent i is defined as follows:

$$B_i(p) = \left\{x_i \in L_i : \sum_{\omega\in\Omega} p(\omega)\cdot x_i(\omega) \leq \sum_{\omega\in\Omega} p(\omega)\cdot e_i(\omega)\right\}.$$

Definition 1. An allocation x is said to be a **maximin expectations equilibrium** allocation for the economy $\mathcal{E}$, if there exists a price vector p such that for any agent $i \in I$,

1. x_i maximizes $V_i(\cdot)$ subject to the budget set $B_i(p)$;
2. x is feasible.

The following definition of a core concept in the current context implies that coalitions of agents cannot cooperate to become better off in terms of MEU.

Definition 2. A feasible allocation x is said to be a **maximin core** allocation for the economy $\mathcal{E}$, if there do not exist a coalition $C \subseteq I$, $C \neq \emptyset$, and an allocation $\{y_i \in L_i\}_{i\in C}$ such that

(i) $V_i(y_i) > V_i(x_i)$ for all $i \in C$;
(ii) $\sum_{i\in C} y_i(\omega) = \sum_{i\in C} e_i(\omega)$ for all $\omega \in \Omega$.

The allocation is said to be **maximin efficient** if $C = I$.

Remark 3. The notions of maximin expectations equilibrium, maximin core and maximin efficiency in the above definitions correspond to the concepts of Walrasian equilibrium, core and efficiency in the standard model. If Bayesian expected utilities, instead of maximin expected utilities, are used in Definition 1, and the private information measurability assumption is imposed on allocations, then the solution concept is Walrasian expectations equilibrium defined in Radner (1968, 1982). In particular, the Walrasian expectations equilibrium is defined as follows: an allocation $x = (x_1, \ldots, x_s)$ is said to be a **Walrasian expectations equilibrium** allocation for the economy $\mathcal{E}$, if x_i is an $\mathcal{F}_i$-measurable mapping for each agent i and there exists a price vector p such that for any agent $i \in I$,

1. x_i maximizes agent i's expected utility subject to the budget set $B_i(p)$;
2. $\sum_{i\in I} x_i \leq \sum_{i\in I} e_i$.

The following example shows that MEE provides strictly higher efficiency than the (free disposal) WEE allocations. Furthermore, we show that the MEE is incentive compatible.

Example 1.[15] Consider the following economy with one commodity, the agent space is $I = \{1, 2\}$ and the state space is $\Omega = \{a, b, c\}$. The initial endowments and information partitions of agents are given by

$$e_1 = (5, 5, 0), \qquad \Pi_1 = \{\{a, b\}, \{c\}\};$$
$$e_2 = (5, 0, 5), \qquad \Pi_2 = \{\{a, c\}, \{b\}\}.$$

[15] This example has been analyzed in Glycopantis and Yannelis (2005) in Bayesian preference setting for the existence and incentive compatibility of Walrasian expectations equilibrium and private core, and in Liu and Yannelis (2013) in maximin preference setting for the existence and incentive compatibility of maximin core. See also Bhowmik et al. (2014).

It is also assumed that for $i \in I$, $u_i(\omega, x_i) = \sqrt{x_i}$, which is strictly concave and monotone in x_i, and the priors for both agents are the same: $\mu(\{\omega\}) = \frac{1}{3}$ for every $\omega \in \Omega$.

Suppose that agents are both Bayesian expected utility maximizers. It can be easily checked that there is no (non-free disposal) WEE. If we allow for free disposal, $x_1 = (4, 4, 1)$ and $x_2 = (4, 1, 4)$ is a (free disposal) WEE allocation with the equilibrium price $p(a) = 0$ and $p(b) = p(c) = \frac{1}{2}$. However, this allocation is not incentive compatible (see Example 2 in Section 5 for details).

If $P_i = \mathcal{M}_i$ for each i, and agents are maximin expected utility maximizers, then there exists an MEE (y, p), where $y_1 = (5, 4, 1)$, $y_2 = (5, 1, 4)$ and $p(a) = 0$, $p(b) = p(c) = \frac{1}{2}$.

If state b or c realizes, the ex post utility of agent 1 will be the same in both Bayesian preference setting and maximin preference setting, since $x_1(b) = y_1(b)$ and $x_1(c) = y_1(c)$. But if state a occurs, the ex post utility of agent 1 with maximin preference will be strictly higher than that in the Bayesian preference setting, since

$$x_1(a) = 4 < 5 = y_1(a).$$

Therefore, the maximin preference allows agents to reach higher efficiency.

The following lemma is standard, which shows that the set of maximin expectations equilibrium allocations is included in the set of maximin core allocations.

Lemma 1. *The set of MEE allocations is a subset of the MC allocations, and hence any maximin expectations equilibrium allocation is maximin efficient.*

This inclusion can be strict. It is clear that both the Arrow–Debreu 'state contingent model' and the deterministic general equilibrium model are special cases of our model: if $\mathcal{F}_i = \mathcal{F} = 2^{\Omega}$ for every $i \in I$, then the maximin expected utility coincides with the Bayesian expected utility and $\mathcal{E}$ is indeed the state contingent model; if Ω is a singleton, then $\mathcal{E}$ is the deterministic model. Moreover, it is well known that in those two models, the set of core allocations could strictly contain the set of Walrasian equilibrium allocations.

We now turn to the issue of the existence of MEE.

Theorem 1. *For an ambiguous asymmetric information economy $\mathcal{E}$, if Assumptions (E) and (U) hold, then there exists an MEE.*

Proof. See Appendix. □

Based on Theorem 1 and Lemma 1, it is straightforward to show that the set of maximin core allocations is also nonempty.

Corollary 1. *Under the conditions of Theorem 1, a maximin core allocation exists.*

3.2. Equivalence theorems

For the economy $\mathcal{E}$, Hervés-Beloso et al. (2005b) provided two equivalence results for the Walrasian expectations equilibrium in terms of the private blocking power of the grand coalition, and Bhowmik and Cao (2013a) extended this result to an asymmetric information economy whose commodity space is a Banach lattice. We will follow this approach and characterize the maximin expectations equilibrium. The two theorems below correspond to Theorems 4.1 and 4.2 of Hervés-Beloso et al. (2005b). The proofs are omitted since the same argument can be followed here.

For an allocation $x = \{x_i\}_{i\in I}$ and a vector $a = (a_1, \dots, a_s) \in [0, 1]^s$, consider the ambiguous asymmetric information economy $\mathcal{E}(a, x)$ which is identical with $\mathcal{E}$ except for the random initial endowment of each agent i given by the convex combination $e_i(a_i, x_i) = a_i e_i + (1 - a_i)x_i$.

Definition 3. An allocation z is **maximin dominated** (or **maximin blocked** by the grand coalition) in the economy $\mathcal{E}(a, x)$ if there exists a feasible allocation y in $\mathcal{E}(a, x)$ such that $V_i(y_i) > V_i(z_i)$ for every $i \in I$.

Theorem 2. *The allocation x is an MEE in $\mathcal{E}$ if and only if x is not a maximin dominated allocation in every economy $\mathcal{E}(a, x)$.*

Definition 4. A coalition $S \subseteq I$ maximin blocks an allocation x in the sense of Aubin via $y = \{y_i\}_{i\in S}$ if for all $i \in S$, there is some $\alpha_i \in (0, 1]$ such that $V_i(y_i) > V_i(x_i)$ and $\sum_{i\in S} \alpha_i y_i \leq \sum_{i\in S} \alpha_i e_i$. The **Aubin maximin core** is the set of all feasible allocations that cannot be maximin blocked by any coalition in the sense of Aubin. An allocation x is called Aubin non-dominated if x is not maximin blocked by the grand coalition in the sense of Aubin.

Theorem 3. *The allocation x is an MEE in $\mathcal{E}$ if and only if x is not a maximin dominated allocation in the sense of Aubin in the economy $\mathcal{E}$.*

4. A continuum approach

4.1. Basics

In this section, we introduce the maximin expectations equilibrium and maximin core for an atomless economy. Let the atomless probability space $(T, \mathcal{T}, \lambda)$ denote the agent space. We define an **atomless ambiguous asymmetric information economy** as follows:

$$\mathcal{E}_0 = \{(\Omega, \mathcal{F}); (\mathcal{F}_t, u_t, e_t, \pi_t) : t \in T\}.$$

An **allocation** in the continuum economy $\mathcal{E}_0$ is a mapping f from $T \times \Omega$ to $\mathbb{R}^l_+$ such that $f(\cdot, \omega)$ is integrable for every $\omega \in \Omega$ and $f(t, \cdot) \in l^\infty_+$ for λ-almost all $t \in T$. The allocation is said to be **feasible** if $\int_T f(t, \omega)\, d\lambda(t) = \int_T e(t, \omega)\, d\lambda(t)$ for every $\omega \in \Omega$.

A coalition in T is a measurable set $S \in \mathcal{T}$ such that $\lambda(S) > 0$. An allocation f is **maximin blocked** by a coalition S in the economy $\mathcal{E}_0$ if there exists some $g : S \times \Omega \to \mathbb{R}^l_+$ such that $\int_S g(t, \omega)\, d\lambda(t) = \int_S e(t, \omega)\, d\lambda(t)$ for every $\omega \in \Omega$, and $V_t(g(t)) > V_t(f(t))$ for λ-almost every $t \in S$.

Definition 5. An allocation f is said to be the **maximin core** for the economy $\mathcal{E}_0$ if it is not maximin blocked by any coalition.

Definition 6. An allocation f is said to be a **maximin expectations equilibrium** allocation for the economy $\mathcal{E}_0$, if there exists a price vector p such that

1. f_t maximizes $V_t(\cdot)$ subject to the budget set $B_t(p)$ for λ-almost all $t \in T$;
2. f is feasible.

4.2. A continuum interpretation of the finite economy

We associate an atomless economy $\mathcal{E}_c$ with the discrete economy $\mathcal{E}$ as in García-Cutrín and Hervés-Beloso (1993), Hervés-Beloso et al. (2005a,b) and Bhowmik and Cao (2013a). The space of agents in $\mathcal{E}_c$ is the Lebesgue unit interval $(T, \mathcal{T}, \mu)$ such that $T = \cup_{i=1}^s T_i$, where $T_i = [\frac{i-1}{s}, \frac{i}{s})$ for $i = 1, \dots, s-1$ and $T_s = [\frac{s-1}{s}, 1]$. For each agent $t \in T_i$, set $\mathcal{F}_t = \mathcal{F}_i$, $\pi_t = \pi_i$, $u_t = u_i$ and $e_t = e_i$. Thus, the maximin ex ante utility V_t of agent t is V_i. We refer to T_i as the set of agents of type i, and

$$\mathcal{E}_c = \{(\Omega, \mathcal{F}); (T, \mathcal{F}_i, V_i, e_i, \pi_i) : i \in I = \{1, \dots, s\}\}$$

is the **economy with the equal treatment property**. The allocations in $\mathcal{E}$ and $\mathcal{E}_c$ are closely related: for any allocation f in $\mathcal{E}_c$, there is a corresponding allocation x in $\mathcal{E}$, where $x_i(\omega) = \frac{1}{\mu(T_i)}$

$\int_{T_i} f(t, \omega) \, d\mu(t)$ for all $i \in I$ and $\omega \in \Omega$; conversely, an allocation x in $\mathcal{E}$ can be interpreted as an allocation f in $\mathcal{E}_c$, where $f(t, \omega) = x_i(\omega)$ for all $t \in T_i$, $\omega \in \Omega$ and $i \in I$. f is said to be a step allocation if $f(\cdot, \omega)$ is a constant function on T_i for any $\omega \in \Omega$ and $i \in I$.

Analogously to the theorems in Hervés-Beloso et al. (2005a,b), the next proposition shows that the maximin expectations equilibrium can be considered equivalent in discrete and continuum approaches.

Proposition 2. *Suppose that Assumption (U) holds. Then we have the following properties:*

- *If (x, p) is an MEE for the economy $\mathcal{E}$, then (f, p) is the MEE for the associated continuum economy $\mathcal{E}_c$, where $f(t, \omega) = x_i(\omega)$ if $t \in T_i$.*
- *If (f, p) is an MEE for the economy $\mathcal{E}_c$, then (x, p) is the MEE for the economy $\mathcal{E}$, where $x_i(\omega) = \frac{1}{\mu(T_i)} \int_{T_i} f(t, \omega) \, d\mu$ for any $\omega \in \Omega$.*

The proof is straightforward, interested readers may refer to Theorem 3.1 of Hervés-Beloso et al. (2005b).

4.3. Core equivalence with a countable number of states

The core–Walras equivalence theorem has been recently extended to a Bayesian asymmetric information economy. Specifically, Einy et al. (2001) showed that the Walrasian expectations equilibrium is equivalent to the private core for atomless economies with a finite number of commodities in a free disposal setting, Angeloni and Martins-da-Rocha (2009) completed the discussion by proposing appropriate conditions which guarantees the core equivalence result in non-free disposal context. Hervés-Beloso et al. (2005a,b) and Bhowmik and Cao (2013a) followed the Debreu–Scarf approach and showed that the set of Walrasian expectations equilibrium allocations coincides with the private core in the asymmetric information economy with the equal treatment property, finitely many states and infinitely many commodities.

However, all these discussions focus on the asymmetric information economy with Bayesian expected utilities and a finite state space. Our aim here is to examine whether this result is still true when agents are ambiguous (have maximin expected utilities) and the state space is countable. The theorems below show that the core equivalence theorem holds with either of the following conditions:

1. Maximin expected utility and finitely many states;
2. Maximin expected utility, countably many states and the equal treatment property holds.

Theorem 4. *Let Ω be finite in the atomless economy $\mathcal{E}_0$. Assume that (E) and (U) hold. Then the set of MC allocations coincides with the set of MEE allocations.*

We omit the proof since it is standard, interested readers may check that the proof of the core equivalence theorem in Hildenbrand (1974) with minor modifications still holds.

Theorem 5. *Suppose Assumptions (E) and (U) hold. Let the step allocation f be feasible in the associated continuum economy $\mathcal{E}_c$. Then f is an MEE allocation if and only if f is an MC allocation.*

Proof. See Appendix. □

4.4. An extension of Vind's theorem

Hervés-Beloso et al. (2005a,b) and Bhowmik and Cao (2013a) extended Vind's theorem to an asymmetric information economy with the equal treatment property. Sun and Yannelis (2007) established this theorem in an economy with a continuum of agents and negligible asymmetric information.[16] Below, we extend this result to the atomless ambiguous asymmetric information economy with a countable number of states of nature.

Theorem 6. *Suppose that Assumptions (E) and (U) hold. If the feasible step allocation f is not in the MC of the associated continuum economy $\mathcal{E}_c$, then for any α, $0 < \alpha < 1$, there exists a coalition S such that $\mu(S) = \alpha$, which maximin blocks f.*

Proof. See Appendix. □

5. Efficiency and incentive compatibility under ambiguity

In this section, we will define a notion of maximin incentive compatibility, and then prove that any maximin efficient allocation is maximin incentive compatible.

First, we illustrate the incentive compatibility issue when agents adopt Bayesian preferences.

Example 2 (Example 1 *with Bayesian Preference*)**.** Recall Example 1 in Section 3.1: the agent space is $I = \{1, 2\}$ and the state space is $\Omega = \{a, b, c\}$. The initial endowments and information partitions of agents are given by

$$e_1 = (5, 5, 0), \, \Pi_1 = \{\{a, b\}, \{c\}\};$$
$$e_2 = (5, 0, 5), \, \Pi_2 = \{\{a, c\}, \{b\}\}.$$

It is also assumed that for $i \in I$, $u_i(\omega, x_i) = \sqrt{x_i}$, which is strictly concave and monotone in x_i, and the priors for both agents are the same: $\mu(\{\omega\}) = \frac{1}{3}$ for every $\omega \in \Omega$.

Suppose that agents are Bayesian expected utility maximizers, and all allocations are required to be private information measurable. The no-trade allocation $x_1 = (5, 5, 0)$ and $x_2 = (5, 0, 5)$ is in the private core and it is incentive compatible. Indeed, it has been shown in Koutsougeras and Yannelis (1993) that private core allocations are always CBIC provided that the utility functions are monotone and continuous.

This conclusion is not true in free disposal economies. Glycopantis and Yannelis (2005) pointed out that private core and Walrasian expectations equilibrium allocations need not be incentive compatible in an economy with free disposal. In this example, $x_1 = (4, 4, 1)$ and $x_2 = (4, 1, 4)$ is a (free disposal) WEE allocation with the equilibrium price $p(a) = 0$ and $p(b) = p(c) = \frac{1}{2}$, and hence in the (free disposal) private core. However, this allocation is not incentive compatible. Indeed, if agent 1 observes $\{a, b\}$, he has an incentive to report state c to become better off. Note that agent 2 cannot distinguish the state a from the state c. In particular, if state a occurs, agent 1 has an incentive to report state c because his utility is $u_1(e_1(a) + x_1(c) - e_1(c))$, which is greater than the utility $u_1(x_1(a))$ when he truthfully reports state a. That is,

$$\begin{aligned} u_1(e_1(a) + x_1(c) - e_1(c)) &= u_1(5 + 1 - 0) \\ &= \sqrt{6} > \sqrt{4} = u_1(x_1(a)). \end{aligned}$$

Hence, the free disposal WEE allocation is not incentive compatible.

Note that in the above example, when agent 1 reports $\{c\}$ and agent 2 reports $\{b\}$, there will be incompatible reports. To rule out such situations, we make the following assumption.

[16] For some recent results, see Bhowmik (forthcoming) and references therein.

Assumption (R). For any $i \in I$ and $E_i \in \Pi_i$, $\cap_{i\in I} E_i = \{\omega\}$ for some $\omega \in \Omega$.

Remark 4. This assumption is only needed in this section. Assumption (R) above guarantees that there are no incompatible reports. The assumption that the intersection is a singleton set is without loss of generality. If $\{a, b\} \subseteq \cap_{i\in I} E_i$ for two states a and b, then no one can distinguish these two states and hence they can be combined as one state.

de Castro and Yannelis (2013) showed that their choice of maximin expected utility is both sufficient and necessary for the incentive compatibility of maximin Pareto efficient allocations. In this section, we shall adopt the maximin expected utility considered in de Castro and Yannelis (2013). That is, as in Remark 1, for any two allocations $x_i, y_i \in L_i$, agent i prefers the allocation x_i to the allocation y_i if

$$\sum_{E_i\in\Pi_i}[\inf_{\omega\in E_i} u_i(\omega, x_i(\omega))]\pi_i(E_i) \geq \sum_{E_i\in\Pi_i}[\inf_{\omega\in E_i} u_i(\omega, y_i(\omega))]\pi_i(E_i).$$

Below, we propose a notion of maximin incentive compatibility.

Definition 7. An allocation x is said to be **maximin incentive compatible** (MIC) if the following does not hold:

1. there exists an agent $i \in I$, and two events $E_i^1, E_i^2 \in \Pi_i$;
2. $e_i(\omega) + x_i(b(\omega)) - e_i(b(\omega)) \in \mathbb{R}^l_+$ for each $\omega \in E_i^1$ and $\{b(\omega)\} = \left(\cap_{j\neq i} \Pi_j(\omega)\right) \cap E_i^2$;
3. $$\inf_{\omega_1\in E_i^1} u_i(\omega_1, y_i(\omega_1)) > \inf_{\omega_1\in E_i^1} u_i(\omega_1, x_i(\omega_1)),$$
   where
   $$y_i(\omega) = \begin{cases} e_i(\omega) + x_i(b(\omega)) - e_i(b(\omega)), & \text{if } \omega \in E_i^1; \\ x_i(\omega), & \text{otherwise.} \end{cases}$$

In other words, an allocation is maximin incentive compatible if it is impossible for any agent to misreport the realized event and become better off. That is, if the true event is E_i^1 and agent i reports E_i^2, then the allocation y_i under the misreported event E_i^2 will not make him better off.

In this paper, we consider a partition model for the information structure. Alternatively, one can also consider a type model.

Let $\Omega = \Theta = \prod_{i\in I} \Theta_i$, where Θ_i is the private information set of agent i. For any state $\omega \in \Omega$, $\omega = (\theta_1, \theta_2, \ldots, \theta_s)$, let $\Pi_i(\omega) = \{\theta_i\} \times \Theta_{-i}$, where Θ_{-i} is the set of states for all agents other than i. Then the maximin incentive compatibility can be described as follows, and Definitions 7 and 8 are equivalent.

Definition 8. An allocation x is MIC if for every agent i and two distinct points $\tilde{\theta}_i, \hat{\theta}_i$ in Θ_i such that for every $\theta_{-i} \in \Theta_{-i}$,

$$y_i^{\tilde{\theta}_i}(\hat{\theta}_i, \theta_{-i}) = e_i(\tilde{\theta}_i) + x_i(\hat{\theta}_i, \theta_{-i}) - e_i(\hat{\theta}_i) \in \mathbb{R}^l_+$$

and

$$\inf_{\theta_{-i}\in\Theta_{-i}} u_i(\tilde{\theta}_i, x_i(\tilde{\theta}_i, \theta_{-i})) \geq \inf_{\theta_{-i}\in\Theta_{-i}} u_i(\tilde{\theta}_i, y_i^{\tilde{\theta}_i}(\hat{\theta}_i, \theta_{-i})).$$

Thus, an agent i cannot become better off in terms of maximin expected utility by reporting $\hat{\theta}_i$ when his true state is $\tilde{\theta}_i$.

The following theorem shows that any maximin efficient allocation is maximin incentive compatible.

Theorem 7. *If Assumptions (E), (R) and (U) hold, then any maximin efficient allocation in $\mathcal{E}$ is MIC.*

Proof. See Appendix. □

Corollary 2. *Under the conditions of Theorem 7, any MC or MEE allocation is maximin incentive compatible.*

Remark 5. There is a substantial literature on the mechanism design under ambiguity; see, for example, Bodoh-Creed (2012), de Castro and Yannelis (2013) and Bose and Renou (2014). Bodoh-Creed (2012) considers a standard mechanism design environment except that agents are ambiguity averse with preferences of the maximin expected utility. In particular, Bodoh-Creed (2012) assumes that each agent knows his valuation but has ambiguous beliefs about the distribution of valuations of the other agents which can be modeled by a convex set of priors, while we consider the particular case that this set contains all possible priors. There are significant differences between Bodoh-Creed's paper and ours. In particular, Bodoh-Creed (2012) focuses on the payoff equivalence theorem and characterizes the revenue maximizing mechanism, which could be constrained efficient (*i.e.*, second best efficient). On the contrary, we study the issue between the first best efficiency and incentive compatibility.

Remark 6. One could extend the result of Angelopoulos and Koutsougeras (2015) on maximin value allocations to an ambiguous asymmetric information economy with countably many states. By standard arguments, one could show that the maximin value allocation is maximin efficient, and therefore, it is maximin incentive compatible by the above corollary.

6. Concluding remarks

We presented a new asymmetric information economy framework, where agents face ambiguity (*i.e.*, they are MEU maximizers) and also the state space is not necessarily finite. This new set up allowed us to derive new core–Walras existence and equivalence results. It should be noted that contrary to the Bayesian asymmetric information economy framework, our core and Walrasian equilibrium concepts formulated in an ambiguous asymmetric information economy framework are now incentive compatible and obviously efficient. For this reason, we believe that our new results will be useful to other fields in economics.

We would like to conclude by saying that the continuum of states and modeling perfect competition as in Sun and Yannelis (2007, 2008), Sun et al. (2012, 2013) and Qiao et al. (forthcoming), or modeling the idea of informational smallness (*i.e.*, approximate perfect competition) in countable replica economies as in McLean and Postlewaite (2003), or characterizing cores in economies where agents' information can be altered by coalitions as in Hervés-Beloso et al. (2014) in the presence of ambiguity remain open questions and further research in this direction seems to be needed.

Appendix

A.1. Proof of Proposition 1

It is clear that V_i is increasing and concave, we first show that it is weak* lower semicontinuous.

Suppose that the sequence $\{z^k\}_{k\geq 0} \subseteq L_i$, and $z^k \to z^0$ in the weak* topology as $k \to \infty$. Fix $\epsilon > 0$. Since $z^0 \in L_i = l^\infty_+$, there exists some positive number $K_0 > 0$ such that $z^0(\omega, j) < K_0$ for each $1 \leq j \leq l$ and $\omega \in \Omega$. By Assumption (U.3), there exists some $K > 0$ such that $u_i(\omega, z^0(\omega)) \leq K$ for any $\omega \in \Omega$.

Suppose that $\Pi_i = \{E_m\}_{m\in\mathbb{N}}$. Then there exists some m_0 sufficiently large such that $\pi_i(\cup_{1\le m\le m_0} E_m) > 1 - \frac{\epsilon}{2K}$. Let $\Omega^{m_0} = \cup_{1\le m\le m_0} E_m$. Then we have

$$V_i(z^k) - V_i(z^0) = \inf_{\mu\in P_i} \sum_{\omega\in\Omega} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i} \sum_{\omega\in\Omega} u_i(\omega, z^0(\omega))\mu(\omega)$$
$$\ge \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) - \inf_{\mu\in P_i} \sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega).$$

For the third term, we have

$$\inf_{\mu\in P_i} \sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) \le K\pi_i(\Omega \setminus \Omega^{m_0}) < \frac{\epsilon}{2}.$$

Since z^k weak* converges to z^0 and Ω^{m_0} is finite, $z^k(\omega)$ converges to $z^0(\omega)$ for each $\omega \in \Omega^{m_0}$. Thus, we have

$$\left| \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) \right| < \frac{\epsilon}{2}$$

for k sufficiently large. As a result, $V_i(z^k) - V_i(z^0) > -\epsilon$ for k sufficiently large, which implies that $V_i(\cdot)$ is weak* lower semicontinuous.

Next we show that V_i is continuous in the sup-norm topology. The proof is similar as the argument above.

Suppose that the sequence $\{z^k\}_{k\ge 0} \subseteq L_i$, and $z^k \to z^0$ in the sup-norm topology. Then $\{z^k\}_{k\ge 0}$ is uniformly bounded by some K_0. By Assumption (U.3), there exists some $K > 0$ such that $u_i(\omega, z^k(\omega)) \le K$ for any $k \ge 0$ and $\omega \in \Omega$. Following an analogous argument as in the proof of the weak* lower semicontinuity, one can obtain a finite subset Ω^{m_0} such that $\pi_i(\Omega^{m_0}) > 1 - \frac{\epsilon}{2K}$. Then we have

$$\left|V_i(z^k) - V_i(z^0)\right| = \left| \inf_{\mu\in P_i} \sum_{\omega\in\Omega} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i} \sum_{\omega\in\Omega} u_i(\omega, z^0(\omega))\mu(\omega) \right|$$
$$\le \left| \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) \right| + \sup_{\mu\in P_i} \sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) + \sup_{\mu\in P_i} \sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega).$$

As in the above argument,

$$\sup_{\mu\in P_i} \sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega), \quad \sup_{\mu\in P_i} \sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) < \frac{\epsilon}{2};$$

and

$$\left| \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i} \sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) \right| < \frac{\epsilon}{2}$$

for k sufficiently large. As a result, $\left|V_i(z^k) - V_i(z^0)\right| \le \epsilon$ for k sufficiently large, which implies that $V_i(\cdot)$ is continuous in the sup-norm topology.

A.2. Proofs in Sections 3 and 4

One can view an ambiguous asymmetric information economy $\mathcal{E}$ as a complete information economy $\mathcal{E}_d = \{(l^\infty_+, V_i, e_i) : i \in I\}$ with the agent space I.[17] That is, each agent i has the utility function V_i and the infinite dimensional commodity space l^∞_+. Given the initial endowment $e_i : \Omega \to \mathbb{R}^l_+$ in the economy $\mathcal{E}$, since Ω is countable, e_i can be viewed as a point in the infinite dimensional commodity space l^∞_+ of the deterministic economy $\mathcal{E}_d$. By Proposition 1, the utility function V_i is increasing, concave and norm continuous, and lower semicontinuous in the weak* topology.

Given an allocation $x = (x_1, \ldots, x_s) \in l^\infty_+$ and a price $p \in (l^\infty)^\circ$, for any agent $i \in I$,

$$p \cdot x_i = \int_\Omega x_i(\omega) p(\mathrm{d}\,\omega).$$

An equilibrium in $\mathcal{E}_d$ is a pair $(x = (x_1, \ldots, x_s), p)$ with $x_i \in l^\infty_+$ for each $i \in I$ and $p \in (l^\infty)^\circ$ such that

1. $x_i \in B_i(p) = \{y \in l^\infty_+ : p \cdot y \le p \cdot e_i\}$;
2. x_i maximizes $V_i(\cdot)$ on the budget set $B_i(p)$;
3. $\sum_{i\in I} x_i = \sum_{i\in I} e_i$.

It can be easily checked that if $p \in l^1$, then the equilibrium (x, p) in the economy $\mathcal{E}_d$ is also an MEE in the ambiguous asymmetric information economy $\mathcal{E}$.

Since V_i is norm continuous, it is Mackey continuous with respect to the Mackey topology $\tau(l^\infty, (l^\infty)^\circ)$ by Corollary 6.23 in Aliprantis and Border (2006). Then the economy $\mathcal{E}_d$ has a competitive equilibrium (x^*, p^*) by Propositions 5.2.3 and 5.3.1 in Florenzano (2003), where $p^* \in (l^\infty)^\circ$. Since V_i is lower semicontinuous in the weak* topology, it is also lower semicontinuous in the Mackey topology $\tau(l^\infty, l^1)$. By Theorem 2 in Bewley (1972), we know that p^* is indeed in l^1. One can then normalize p^* such that $\|p^*\|_1 = 1$. Then it is clear that (x^*, p^*) is also a maximin expectations equilibrium in the ambiguous asymmetric information economy $\mathcal{E}$, which proves Theorem 1.

If $\mathcal{E}_c$ is an atomless ambiguous asymmetric information economy, one can also view $\mathcal{E}_c$ as an atomless complete information economy $\mathcal{E}_{cd}$ as above. Then Theorems 5 and 6 follow from Theorems 3.2 and 3.3 in Hervés-Beloso et al. (2005b).

A.3. Proof of Theorem 7

Recall that for any agent i, allocation $z \in L_i$ and event $E \in \Pi_i$, $v_i(E, z) = \inf_{\omega\in E} u_i(\omega, z(\omega))$. Let $\{x_i\}_{i\in I}$ be a maximin efficient allocation, and assume that it is not maximin incentive compatible. Then there exist an agent $i \in I$, and two events $E_i^1, E_i^2 \in \Pi_i$ such that

$$v_i(E_i^1, y_i) > v_i(E_i^1, x_i),$$

where

$$y_i(\omega) = \begin{cases} e_i(\omega) + x_i(b) - e_i(b), \\ \quad \text{if } \omega \in E_i^1, \{b\} = \left(\cap_{j\ne i} \Pi_j(\omega)\right) \cap E_i^2; \\ x_i(\omega), \quad \text{otherwise.} \end{cases}$$

For each $j \ne i$, define $y_j : \Omega \to \mathbb{R}^l_+$ as follows:

$$y_j(\omega) = \begin{cases} e_j(\omega) + x_j(b) - e_j(b), \\ \quad \text{if } \omega \in E_i^1, \{b\} = \left(\cap_{j\ne i} \Pi_j(\omega)\right) \cap E_i^2; \\ x_j(\omega), \quad \text{otherwise.} \end{cases}$$

It can be easily checked that $\{y_i\}_{i\in I}$ is feasible:

[17] Let l^∞ and l^1 represent the spaces of all bounded sequences and all absolutely summable sequences, respectively. Denote by $(l^\infty)^\circ$ the topological dual space of l^∞.

1. If $\omega \in E_i^1$ and $\{b\} = \left(\cap_{j\neq i} \Pi_j(\omega)\right) \cap E_i^2$, then $\sum_{j\in I} y_j(\omega) = \sum_{j\in I} e_j(\omega) + \sum_{j\in I} x_j(b) - \sum_{j\in I} e_j(b) = \sum_{j\in I} e_j(\omega)$, since $\sum_{j\in I} e_j(b) = \sum_{j\in I} x_j(b)$.
2. If $\omega \notin E_i^1$, then $\sum_{j\in I} y_j(\omega) = \sum_{j\in I} x_j(\omega) = \sum_{j\in I} e_j(\omega)$.

We now show that agent i is better off and all other agents are not worse off if considering the allocation y instead of x.

For agent i, if $\omega \notin E_i^1$, then $v_i(\omega, y_i) = v_i(\omega, x_i)$. In addition, $v_i(E_i^1, y_i) > v_i(E_i^1, x_i)$. Therefore, $V_i(y_i) = \sum_{E_i\in\Pi_i} v_i(E_i, y_i)\pi_i(E_i) > \sum_{E_i\in\Pi_i} v_i(E_i, x_i)\pi_i(E_i) = V_i(x_i)$.

For $j \neq i$ and event E_j, if $\omega \in E_i^1$, then there exists a point $b(\omega) \in E_j \cap E_i^2$ such that $e_j(b(\omega)) = e_j(\omega)$ and $y_j(\omega) = e_j(\omega) + x_j(b(\omega)) - e_j(b(\omega)) = x_j(b(\omega))$. Notice that $u_j(\omega, y_j(\omega)) = u_j(\omega, x_j(b(\omega))) = u_j(b(\omega), x_j(b(\omega)))$. If $\omega \notin E_i^1$, then $y_j(\omega) = x_j(\omega)$. Thus, we have

$$\begin{aligned} v_j(E_j, y_j) &= \min\left(\inf_{\omega\in E_j, \omega\in E_i^1} u_j(\omega, y_j(\omega)), \inf_{\omega\in E_j, \omega\notin E_i^1} u_j(\omega, y_j(\omega))\right) \\ &= \min\left(\inf_{\omega\in E_j, \omega\in E_i^1} u_j(b(\omega), x_j(b(\omega))), \right. \\ &\quad \left. \inf_{\omega\in E_j, \omega\notin E_i^1} u_j(\omega, x_j(\omega))\right) \\ &= \inf_{\omega\in E_j, \omega\notin E_i^1} u_j(\omega, x_j(\omega)) \\ &\geq \inf_{\omega\in E_j} u_j(\omega, x_j(\omega)) \\ &= v_j(E_j, x_j). \end{aligned}$$

Then $V_j(y_j) = \sum_{E_j\in\Pi_j} v_j(E_j, y_j)\pi_j(E_j) \geq \sum_{E_j\in\Pi_j} v_j(E_j, x_j)\pi_j(E_j) = V_j(x_j)$ for all $j \neq i$.

Since $\epsilon y_i \to y_i$ as $\epsilon \to 1$ in $(\mathbb{R}_+^l)^\infty$ and V_i is continuous, there exists $\epsilon \in (0, 1)$ such that

$$V_i(\epsilon y_i) > V_i(x_i) \quad \text{for all } i \in C.$$

For all $\omega \in \Omega$, define

$$z_j(\omega) = \begin{cases} \epsilon y_j(\omega) & \text{if } j = i; \\ y_j(\omega) + \dfrac{1-\epsilon}{\|I-1\|} y_i(\omega) & \text{if } j \neq i. \end{cases}$$

Then $V_i(z_i) = V_i(\epsilon y_i) > V_i(x_i)$. Moreover, since $u_i(\omega, \cdot)$ is strongly monotone, for all $j \neq i$

$$V_j(z_j) = V_j\left(y_j + \frac{1-\epsilon}{\|I-1\|} y_i\right) > V_j(y_j) \geq V_j(x_j). \tag{2}$$

Notice that for every $\omega \in \Omega$,

$$\begin{aligned} \sum_{i\in I} z_i(\omega) &= \epsilon y_i(\omega) + \sum_{j\neq i} y_j(\omega) + (1-\epsilon) y_i(\omega) \\ &= \sum_{i\in I} y_i(\omega) = \sum_{i\in I} e_i(\omega). \end{aligned}$$

That is, z is feasible and by (2), $V_i(z_i) > V_i(x_i)$ for any i. Thus, $\{x_i\}_{i\in I}$ is not maximin efficient, a contradiction.

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